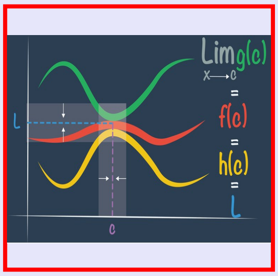
Calculus I

Lecture 1



Feb 19-8:47 AM

Intro. to Functions:

For every input value x, there is only one output value y.

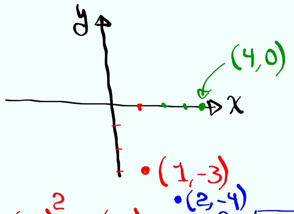
(x, y) ordered-Pair.

Function notation $y = f(x)$

ex: $y = f(x) = x^2 - 4x$

If $x = 1 \rightarrow y = f(1) = 1^2 - 4(1) = 1 - 4 = -3$

$(1, -3)$



find $f(\underset{\uparrow \atop x=2}{2}) = (2)^2 - 4(2) = 4 - 8 = \boxed{-4}$

find $f(4) = 4^2 - 4(4) = 16 - 16 = \boxed{0}$

Do not use Ø for Zero.

Jan 5-8:04 AM

Given $f(x) = x - |x|$

Find $f(0)$, $f(2)$, and $f(-2)$.

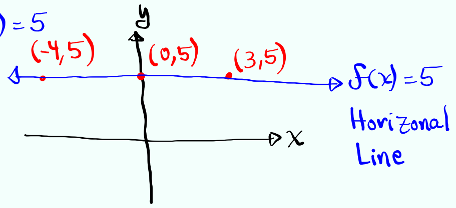
$f(0) = 0 - |0| = 0 - 0 = 0$

$f(2) = 2 - |2| = 2 - 2 = 0$

$f(-2) = -2 - |-2| = -2 - 2 = \boxed{-4}$

Constant Function $f(x) = c$

ex: $f(x) = 5$



$f(0) = 5$

$f(3) = 5$

$f(-4) = 5$

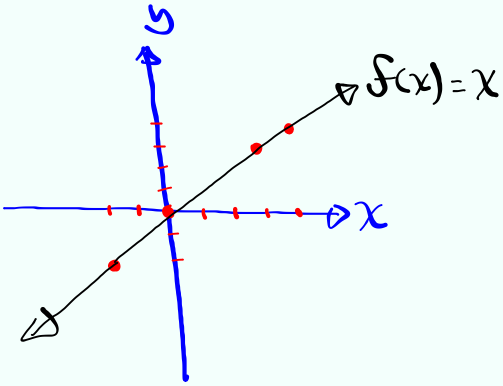
Find $\dfrac{f(x) - f(4)}{x-4} = \dfrac{5 - 5}{x-4} = \dfrac{0}{x-4} = \boxed{0}$ $x \neq 4$

if $x = 4 \rightarrow \dfrac{0}{0}$ Indeterminate form

Jan 5-8:13 AM

Identity Function $f(x) = x \rightarrow I(x) = x$

$f(x) = x$



$f(3) = 3$

$f(-2) = -2$

$f(4) = 4$

$f(0) = 0$

Simplify $\dfrac{f(x) - f(6)}{x-6} = \dfrac{x-6}{x-6} = \boxed{1}$ if $x \neq 6$.

If $x = 6 \rightarrow \dfrac{6-6}{6-6} = \dfrac{0}{0}$ I.F.

Jan 5-8:22 AM

Linear Function $f(x) = mx + b$

Slope

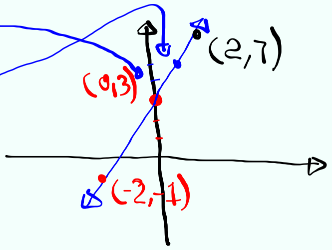
Y-Int (0,b)

ex: $f(x) = 2x + 3$

Slope $m=2$

Y-Int (0,3)

↳ $\frac{\text{Rise}}{\text{Run}} = 2 = \frac{2}{1}$



$f(2) = 2(2) + 3 = \boxed{7}$

$f(-2) = 2(-2) + 3 = \boxed{-1}$

Simplify $\frac{f(x) - f(4)}{x-4} = \frac{2x+3 - 11}{x-4} = \frac{2x-8}{x-4}$

$f(4) = 2(4) + 3 = 11$

$= \frac{2\cancel{(x-4)}}{\cancel{x-4}} = \boxed{2}$ if $x \neq 4$

Jan 5-8:27 AM

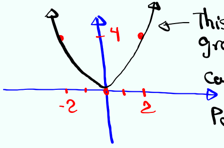
Square Function $f(x) = x^2$

Find $f(-2)$, $f(0)$, and $f(2)$.

$f(-2) = (-2)^2 = 4$

$f(0) = 0^2 = 0$

$f(2) = 2^2 = 4$



This graph is called Parabola

Domain → x → can be any real #

Range → y → $y \geq 0$

Simplify $\frac{f(x) - f(3)}{x-3} = \frac{x^2 - 3^2}{x-3}$

$= \frac{x^2 - 9}{x-3} = \frac{\cancel{(x-3)}(x+3)}{\cancel{x-3}}$

Recall $A^2 - B^2 = (A+B)(A-B)$

$= \boxed{x+3}$ if $x \neq 3$

Simplify $\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - x^2}{h}$

$(x+h)^2 = (x+h)(x+h)$

$= x^2 + xh + hx + h^2$

$= x^2 + 2xh + h^2$

$= \frac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h}$

$= \frac{2xh + h^2}{h}$ if $h \neq 0$

$= \frac{\cancel{h}(2x+h)}{\cancel{h}} = \boxed{2x+h}$

Jan 5-8:35 AM

Square-Root Function $f(x)=\sqrt{x}$

Find $f(0)$, $f(4)$, $f(-4)$, $f(9)$, and $f(-5)$.

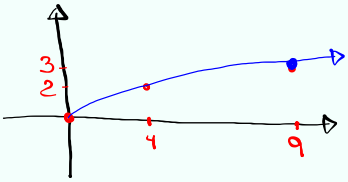
$f(0)=\sqrt{0}=0$

$f(4)=\sqrt{4}=2$

$f(-4)=\sqrt{-4}$ undefined

$f(9)=\sqrt{9}=3$

$f(-5)=\sqrt{-5}$ undefined



Domain → $x\geq 0$ → $[0,\infty)$

Range → $y\geq 0$ → $[0,\infty)$

Interval notation

Simplify $\dfrac{f(x)-f(4)}{x-4}$

$$=\frac{\sqrt{x}-2}{x-4}=\frac{(\sqrt{x}-2)(\sqrt{x}+2)}{(x-4)(\sqrt{x}+2)}=\frac{\sqrt{x^2}\cancel{+2\sqrt{x}}\cancel{-2\sqrt{x}}-4}{(x-4)(\sqrt{x}+2)}$$

Rationalize the numerator. Multiply by the conjugate of the numerator.

$$=\frac{\cancel{x-4}}{\cancel{(x-4)}(\sqrt{x}+2)}=\boxed{\frac{1}{\sqrt{x}+2}}$$

Jan 5-8:51 AM

Absolute-Value Function $f(x)=|x|$

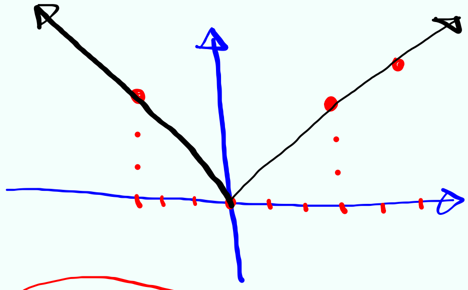
Find $f(-3)$, $f(0)$, $f(3)$, and $f(5)$.

$f(-3)=|-3|=3$

$f(0)=|0|=0$

$f(3)=3$

$f(5)=5$



$$|x|=\begin{cases}-x & \text{if } x<0\\ x & \text{if } x\geq 0\end{cases}$$

$f(-4)=-(-4)=4$

$f(6)=6$

Jan 5-9:04 AM

Graph $f(x) = x + |x|$

If $x \geq 0 \rightarrow f(x) = x + x = 2x$

If $x < 0 \rightarrow f(x) = x - x = 0$

$$f(x) = \begin{cases} 0 & \text{if } x < 0 \\ 2x & \text{if } x \geq 0 \end{cases}$$

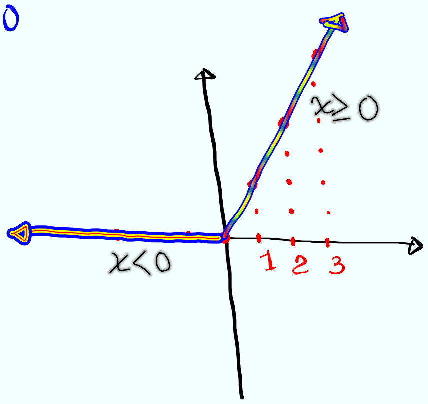
Piece-wise Function

$f(-1) = 0$

$f(1) = 2(1) = 2$

$f(-3) = 0$

$f(3) = 2(3) = 6$



Jan 5-9:10 AM

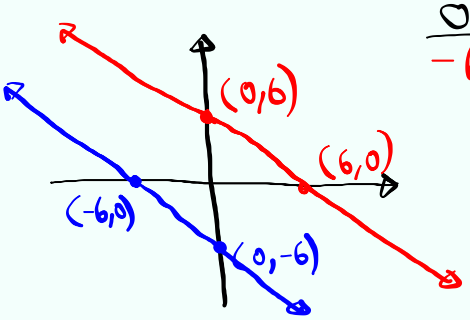
Graph $|x + y| = 6$

Abs. value of what is equal to 6? 6 or -6

$x + y = 6$ OR $x + y = -6$

x	y
0	6
6	0

x	y
0	-6
-6	0



Jan 5-9:16 AM

Reciprocal Function $f(x)=\frac{1}{x}$, $x\neq 0$

Find $f(1)$, $f(3)$, and $f(\frac{-1}{4})$

Can x be any number? NO

$f(1)=\frac{1}{1}=1$

Domain $x\neq 0$

$(-\infty,0)\cup(0,\infty)$

↑

union

OR

$f(3)=\frac{1}{3}$

$$f\left(\frac{-1}{4}\right)=\frac{1}{\frac{-1}{4}}=1\div\frac{-1}{4}=1\cdot\frac{-4}{1}=\boxed{-4}$$

Simplify $\frac{f(x)-f(2)}{x-2}=\frac{\frac{1}{x}-\frac{1}{2}}{x-2}$

Hint: Multiply top & bottom by LCD=2x.

$$=\frac{2x\left(\frac{1}{x}-\frac{1}{2}\right)}{2x(x-2)}=\frac{\cancel{2x}\cdot\frac{1}{\cancel{x}}-\cancel{2}x\cdot\frac{1}{\cancel{2}}}{2x(x-2)}=\frac{\overset{-1}{\cancel{2-x}}}{2x\cancel{(x-2)}}$$

Recall $\frac{a-b}{b-a}=-1$

$$=\boxed{\frac{-1}{2x}}$$

Jan 5-9:20 AM

Simplify

1) $(x+3)^2-(x-3)^2$

$=(x+3)(x+3)-(x-3)(x-3)$

$=x^2+3x+3x+9-(x^2-3x-3x+9)$

$=\cancel{x^2}+6x\cancel{+9}\cancel{-x^2}+6x\cancel{-9}=\boxed{12x}$

$(A+B)(A-B)=A^2-B^2$

2) $(x^2+9)(x+3)(x-3)$

$=(x^2+9)(x^2-3^2)$

$=(x^2+9)(x^2-9)=(x^2)^2-(9)^2=\boxed{x^4-81}$

Jan 5-9:32 AM

Class QZ 1

Given $f(x) = 2x - 4$

Box Your Final Answer

1) Find $f(0)$. $f(0)=2(0)-4 = 0-4 = \boxed{-4}$

2) Solve $f(x)=0$.

$$2x-4=0$$
$$2x=4$$
$$x=\frac{4}{2} \quad \boxed{x=2}$$

Jan 5-9:40 AM

Introduction to limit:

$$\lim_{x\to a} f(x) = L$$

as x gets close to a

$f(a)$ gets close to L.

Always plug it in, and evaluate (if lucky)

Evaluate $\lim_{\underset{a}{x\to 4}} (\overset{f(x)}{x^2-2x}) = 4^2-2(4)$

$= 16-8$

$= \boxed{8}$

Evaluate $\lim_{x\to 6} \left(\frac{2}{3}x-4\right) = \frac{2}{\cancel{3}}(\overset{2}{\cancel{6}})-4$

$= 2(2)-4$

$= \boxed{0}$

Evaluate $\lim_{x\to \frac{\pi}{2}} (\sin x + \cos x)$

$= \sin\frac{\pi}{2} + \cos\frac{\pi}{2} = 1+0$

$= \boxed{1}$

Recall $\frac{\pi}{2} = 90^\circ$

$\sin 90^\circ = 1$ $\cos 90^\circ = 0$

Jan 5-10:21 AM

Evaluate $\lim_{x\to2}\frac{x^2-5x+6}{x^2-4}$

Plug it in, and hope for the best

$$=\frac{2^2-5(2)+6}{2^2-4}=\frac{4-10+6}{4-4}=\frac{0}{0}\text{ I.F.}$$

when working with Polynomial, use factoring.

$$\lim_{x\to2}\frac{x^2-5x+6}{x^2-4}=\lim_{x\to2}\frac{(x-3)\cancel{(x-2)}}{(x+2)\cancel{(x-2)}}$$

$$=\lim_{x\to2}\frac{x-3}{x+2}=\frac{2-3}{2+2}=\boxed{\frac{-1}{4}}$$

Jan 5-10:29 AM

Evaluate $\lim_{x\to2}\frac{x^3-8}{x^2-6x+8}=\frac{2^3-8}{2^2-6(2)+8}$

$$=\frac{8-8}{4-12+8}=\frac{0}{0}\text{ I.F.}$$

use factoring to proceed

$$\lim_{x\to2}\frac{x^3-\overset{\to 2^3}{8}}{x^2-6x+8}=\lim_{x\to2}\frac{\cancel{(x-2)}(x^2+2x+4)}{(x-4)\cancel{(x-2)}}$$

Recall

$A^3-B^3=(A-B)(A^2+AB+B^2)$

$$=\lim_{x\to2}\frac{x^2+2x+4}{x-4}$$

$$=\frac{2^2+2(2)+4}{2-4}$$

$$=\frac{12}{-2}=\boxed{-6}$$

Jan 5-10:34 AM

Evaluate $\lim_{x\to -2} \frac{x^3+8}{x^2+10x+16} = \frac{(-2)^3+8}{(-2)^2+10(-2)+16} = \frac{-8+8}{4-20+16}$

$= \frac{0}{0}$

I.F.

$= \lim_{x\to -2} \frac{x^3+2^3}{x^2+10x+16} = \lim_{x\to -2} \frac{\cancel{(x+2)}(x^2-2x+4)}{\cancel{(x+2)}(x+8)}$

Recall

$A^3+B^3=(A+B)(A^2-AB+B^2)$

$= \lim_{x\to -2} \frac{x^2-2x+4}{x+8}$

$= \frac{(-2)^2-2(-2)+4}{-2+8} = \frac{12}{6}$

$= \boxed{2}$

Jan 5-10:41 AM

Evaluate $\lim_{x\to 2} \frac{\frac{1}{x}-\frac{1}{2}}{x-2} = \frac{\frac{1}{2}-\frac{1}{2}}{2-2} = \frac{0}{0}$ I.F.

when working with fractions, multiply top & bottom by LCD to simplify. LCD $= 2x$

$= \lim_{x\to 2} \frac{2x\left(\frac{1}{x}-\frac{1}{2}\right)}{2x(x-2)} = \lim_{x\to 2} \frac{2\cancel{x}\cdot\frac{1}{\cancel{x}} - \cancel{2}x\cdot\frac{1}{\cancel{2}}}{2x(x-2)}$

$= \lim_{x\to 2} \frac{2-x}{2x(x-2)} = \lim_{x\to 2} \frac{-1}{2x}$

$= \frac{-1}{2(2)} = \boxed{\frac{-1}{4}}$

Jan 5-10:50 AM

Evaluate $\lim_{x\to 0} \frac{\frac{1}{x+1}-1}{x} = \frac{\frac{1}{0+1}-1}{0} = \frac{\frac{1}{1}-1}{0}$

$= \frac{1-1}{0}$

$= \frac{0}{0}$ I.F.

LCD: $x+1$

$$= \lim_{x\to 0} \frac{(x+1)\left(\frac{1}{x+1}-1\right)}{(x+1)\cdot x} = \lim_{x\to 0} \frac{1-(x+1)}{x(x+1)}$$

$$= \lim_{x\to 0} \frac{\cancel{1}-\cancel{x}-\cancel{1}}{\cancel{x}(x+1)} = \lim_{x\to 0} \frac{-1}{x+1} = \frac{-1}{0+1} = \frac{-1}{1}$$

$$= \boxed{-1}$$

Jan 5-10:55 AM

If we have radicals → we must rationalize.

Evaluate $\lim_{x\to 16} \frac{\sqrt{x}-4}{x-16} = \frac{\sqrt{16}-4}{16-16} = \frac{4-4}{0} = \frac{0}{0}$

I.F.

How do we rationalize the numerator?

Multiply by its Conjugate → $\sqrt{x}+4$

$$\lim_{x\to 16} \frac{\overset{A\ -B}{(\sqrt{x}-4)}\overset{A\ +B}{(\sqrt{x}+4)}}{(x-16)(\sqrt{x}+4)} = \lim_{x\to 16} \frac{\overset{A^2-B^2}{(\sqrt{x})^2-(4)^2}}{(x-16)(\sqrt{x}+4)}$$

$$= \lim_{x\to 16} \frac{\cancel{x-16}}{\cancel{(x-16)}(\sqrt{x}+4)} = \lim_{x\to 16} \frac{1}{\sqrt{x}+4}$$

$$= \dots = \boxed{\frac{1}{8}}$$

Jan 5-11:02 AM

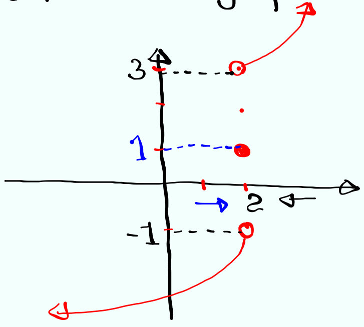
One - Sided limit:

$\lim_{x \to a^+} f(x)$

from the right side

$\lim_{x \to a^-} f(x)$ from the left side.

Consider the graph of f(x) below:



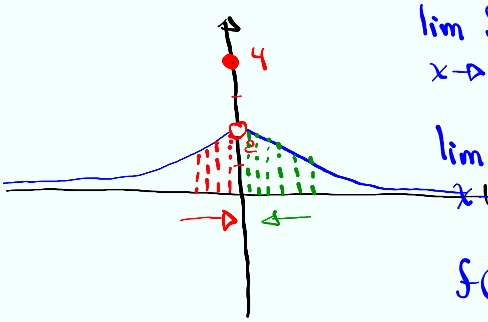
$\lim_{x \to 2^+} f(x) = 3$

$\lim_{x \to 2^-} f(x) = -1$

$f(2) = 1$

Jan 5-11:09 AM

Consider the graph of f(x) below



$\lim_{x \to 0^+} f(x) = 2$

$\lim_{x \to 0^-} f(x) = 2$

$\rightarrow \lim_{x \to 0} f(x) = 2$

$f(0) = \boxed{4}$

Bonus Question

$\lim_{x \to \infty} f(x) = \boxed{0}$

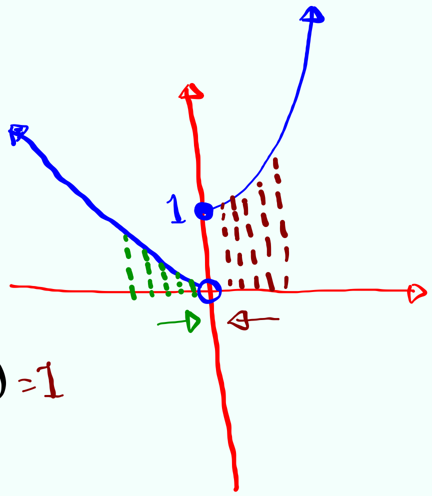
$\lim_{x \to -\infty} f(x) = \boxed{0}$

If $\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x)$, then $\lim_{x \to a} f(x)$ exists.

Jan 5-11:14 AM

Given

$$f(x) = \begin{cases} -x & \text{if } x < 0 \\ x^2 + 1 & \text{if } x \geq 0 \end{cases}$$



$\lim_{x \to 0^-} f(x) = 0$

$\lim_{x \to 0^+} f(x) = 1$

$\lim_{x \to 0} f(x) =$ D.N.E.

$f(0) = 1$

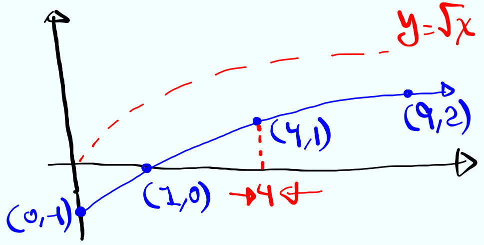
Jan 5-11:21 AM

Consider $f(x) = \sqrt{x} - 1$

1) Find $f(0)$, $f(1)$, $f(4)$, and $f(9)$

$= \boxed{-1}$ $= \boxed{0}$ $= \boxed{1}$ $= \boxed{2}$

2) Graph $f(x)$



$\lim_{x \to 4^+} f(x) = \boxed{1}$

$\lim_{x \to 4^-} f(x) = \boxed{1}$

$\lim_{x \to 4} f(x) = \boxed{1}$

$f(4) = \boxed{1}$

Jan 5-11:26 AM

Given $f(x) = x^2 + x$

1) Find $\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 + (x+h) - (x^2+x)}{h}$

$$= \frac{(x+h)(x+h) + (x+h) - (x^2+x)}{h}$$

$$= \frac{\cancel{x^2} + xh + xh + h^2 + \cancel{x} + h - \cancel{x^2} - \cancel{x}}{h} = \frac{2xh + h^2 + h}{h}$$

$$= \frac{\cancel{h}(2x + h + 1)}{\cancel{h}} = 2x + h + 1$$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \cdots = \lim_{h \to 0} (2x + h + 1) = 2x + 0 + 1 = \boxed{2x+1}$$

Jan 5-11:32 AM

$f(x) = \frac{1}{x}$

Evaluate $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$$= \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

Use LCD $= (x+h)x$ to clear fractions

$$= \lim_{h \to 0} \frac{\cancel{(x+h)} \cdot x \cdot \frac{1}{\cancel{x+h}} - (x+h) \cdot \cancel{x} \cdot \frac{1}{\cancel{x}}}{(x+h) \cdot x \cdot h}$$

$$= \lim_{h \to 0} \frac{x - (x+h)}{(x+h) \cdot x \cdot h} = \lim_{h \to 0} \frac{-\cancel{h}}{(x+h) \cdot x \cdot \cancel{h}}$$

$$= \lim_{h \to 0} \frac{-1}{(x+h) \cdot x} = \frac{-1}{(x+0) \cdot x} = \frac{-1}{x \cdot x} = \boxed{\frac{-1}{x^2}}$$

Jan 5-11:41 AM

$f(x) = x^2 - 2x - 8$

1) Find $f(0) = 0^2 - 2(0) - 8 = 0 - 0 - 8 = \boxed{-8}$

2) Solve $f(x) = 0$. Solve $x^2 - 2x - 8 = 0$

Method I: Factoring $(x+2)(x-4) = 0$

$x+2=0$ $x-4=0$

$\boxed{x=-2}$ $\boxed{x=4}$

Method II: Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-(-2) \pm \sqrt{(-2)^2-4(1)(-8)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{4+32}}{2} = \frac{2 \pm \sqrt{36}}{2} = \frac{2 \pm 6}{2}$$

$x = \frac{2+6}{2} = \frac{8}{2} = \boxed{4}$

$x = \frac{2-6}{2} = \frac{-4}{2} = \boxed{-2}$

Method III: Comp. the Square

$x^2 - 2x + 1 = 8 + 1$

$(x-1)^2 = 9$

$x-1=3$ $x-1=-3$

$\boxed{x=4}$ $\boxed{x=-2}$

Jan 5-11:52 AM

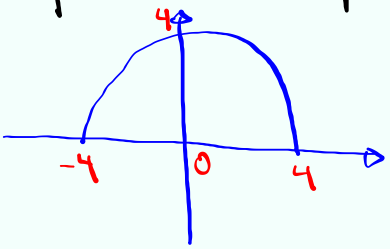
Given $f(x) = \sqrt{16 - x^2}$

1) Replace $f(x)$ with y. $y = \sqrt{16 - x^2}$

2) Square both sides to remove the radical.

$(y)^2 = \left(\sqrt{16 - x^2}\right)^2 \rightarrow y^2 = 16 - x^2$

3) Graph the new equation. $x^2 + y^2 = 16$



Circle

Center (0,0)

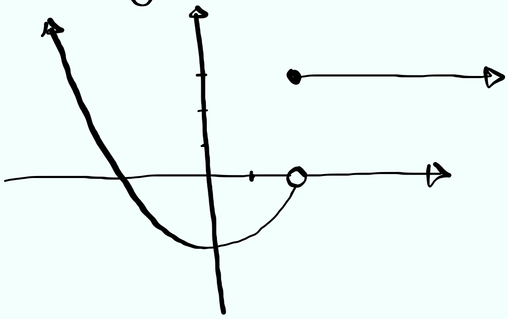
Radius 4

Domain $[-4,4]$ Range $[0,4]$

Jan 5-12:02 PM

Class QZ 2

Use the graph below for $y=f(x)$



1) $\lim_{x\to 2^+} f(x) = \boxed{3}$

2) $\lim_{x\to 2^-} f(x) = \boxed{0}$

3) $\lim_{x\to 2} f(x) = \boxed{\text{D.N.E.}}$

4) $f(2) = \boxed{3}$

Jan 5-12:08 PM